# Markscheme 

May 2019

# Mathematical studies 

## Standard level

## Paper 1

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## Paper 1 Markscheme Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6 .

## 1 Abbreviations

The markscheme may make use of the following abbreviations:
M Marks awarded for Method
A Marks awarded for an Answer or for Accuracy
C Marks awarded for Correct answers (irrespective of working shown)
$\boldsymbol{R} \quad$ Marks awarded for clear Reasoning
ft Marks that can be awarded as follow through from previous results in the question

## 2 Method of Marking

(a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
(b) If the candidate has full marks on a question use the $\boldsymbol{C 6}$ annotation, if the candidate has made an attempt but scores zero marks use C0. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations MUST be shown.
(c) In this paper, if the correct answer is seen on the answer line the maximum mark is awarded. There is no need to check the working! Award $\boldsymbol{C}$ marks and move on.
(d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
(e) If the answer is wrong, marks should be awarded for the working according to the markscheme.
(f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
(g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
(h) If correct working results in a correct answer in the working box but then further working is developed, indicating a lack of mathematical understanding full marks should not be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer as long as it is unambiguous. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

|  | Correct answer seen | Further working seen | Action |
| :--- | :--- | :--- | :--- |
| 1. | $8 \sqrt{2}$ | $5.65685 \ldots$ <br> (incorrect decimal value) | Award the final (A1) <br> (ignore the further working) |
| 2. | $(x-6)(x+1)$ | $x=6$ and -1 | Do not award the final (A1) <br> (see next example) |

Example: Factorise $x^{2}-5 x-6$

| Markscheme | Candidates' Scripts | Marking |  |
| :---: | :--- | :--- | :--- |
| $(x-6)(x+1)$ | (A1)(A1) | (i) | Answer line: $(x+6)(x+1)$ |
| (A0)(A1) |  |  |  |
|  | (ii)Working box: $(x-6)(x+1)$ <br> followed by $x=6$ and -1, or just $6,-1$ <br> in either working box or on answer line. | (A1) |  |

Follow through (ft) Marks
Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, follow through (ft) marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.
(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
(b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final $\boldsymbol{A}$ mark should not be awarded.
(c) If a question is transformed by an error into a different, much simpler question then follow through may not apply.
(d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. The markscheme will clearly indicate where this applies.
(f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry


## Using the Markscheme

(a) $\boldsymbol{A}$ marks are dependent on the preceding $\boldsymbol{M}$ mark being awarded, it is not possible to award (M0)(A1) Once an (M0) has been awarded, all subsequent $\boldsymbol{A}$ marks are lost in that part of the question, even if calculations are performed correctly, until the next $\boldsymbol{M}$ mark.
The only exception will be for an answer where the accuracy is specified in the question - see section 5.
(b) $\boldsymbol{A}$ marks are dependent on the $\boldsymbol{R}$ mark being awarded, it is not possible to award (A1)(R0). Hence the (A1) is not awarded for a correct answer if no reason or the wrong reason is given.
(c) Alternative methods may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by 'OR' etc.
(d) Unless the question specifies otherwise, accept equivalent forms. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$. On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value (for example $\frac{2}{3}$ if applicable) ;
the full calculator display in the form $2.83163 \ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a different 3 significant figure answer, these solutions will also be given.
(e) As this is an international examination, all valid alternative forms of notation should be accepted. Some examples of these are:

Decimal points: $1.7 ; 1{ }^{\prime} 7 ; 1 \cdot 7 ; 1,7$.
Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .
Different descriptions of an interval: $3<x<5$; $(3,5)$; ] 3, 5 [.
Different forms of notation for set properties (e.g. complement): $A^{\prime} ; \bar{A} ; A^{c} ; U-A ;(A ; U \backslash A$.
Different forms of logic notation: $\neg p ; p^{\prime} ; \tilde{p} ; \bar{p} ; \sim p ;-p$.

$$
p \Rightarrow q ; p \rightarrow q ; q \Leftarrow p .
$$

Significance level may be written as $\alpha$.
(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.
Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's answer is seen to 4 sf or greater and would round to the required 3 sf answer, then award (A1) and ignore subsequent rounding.
Note: The unrounded answer may appear in either the working box or on the final answer line.
2. If the candidate's unrounded answer is not seen then award (A1) if the answer given is correctly rounded to 2 or more significant figures, otherwise (AO).
Note: If the candidate's unrounded answer is not seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
3. If a correct 2 sf answer is used in subsequent parts, then working must be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.


## Examples:

| Markscheme |  | Candidates' Scripts |  |  | Marking |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9.43 (9.43398...) | (A1) | (i) $9.43398 \ldots$ is seen in the working box followed by 9; 9.4; 9.43; 9.434 etc. (correctly rounded) <br> (ii) $9.43398 \ldots$ is seen in the working box followed by 9.433; 9.44 etc. (incorrectly rounded) |  |  | (A1) |
|  |  |  |  |  | (A1) |
|  |  | (iii) <br> (iv) | 9.4 |  | (A1) |
|  |  |  | 9 |  | (AO) <br> (correct to 1 sf) |
|  |  | (v) | 9.3 | (incor | (AO) rounded to 2 sf) |
|  |  | (vi) | 9.44 |  | (AO) rounded to 3 sf) |


| Markscheme |  | Candidates' Scripts |  |  | Marking |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7.44 (7.43798...) | (A1) | (i) $7.43798 \ldots$ is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc. (correctly rounded) <br> (ii) $7.43798 \ldots$ is seen in the working box followed by 7.437; 7.43 etc. (incorrectly rounded) |  |  | (A1) |
|  |  |  |  |  | (A1) |
|  |  | (iii) <br> (iv) | 7.4 |  | (A1) |
|  |  |  | 7 |  | (AO) (correct to 1 sf) |
|  |  | (v) | 7.5 |  | (AO) rounded to 2 sf) |
|  |  | (vi) | 7.43 |  | (AO) rounded to 3 sf) |

Example: $\quad \mathrm{ABC}$ is a right angled triangle with angle $\mathrm{ABC}=90^{\circ}, \mathrm{AC}=32 \mathrm{~cm}$ and $\mathrm{AB}=30 \mathrm{~cm}$. Find (a) the length of BC , (b) The area of triangle ABC .

\begin{tabular}{|c|c|c|c|}
\hline Markscheme \& \& Candidates' Scripts \& Marking \\
\hline \begin{tabular}{l}
(a) \(\mathrm{BC}=\sqrt{32^{2}-30^{2}}\) \\
(M1) \\
Award (M1) for correct substitution in Pythagoras' formula
\[
\begin{equation*}
=11.1(\sqrt{124}, 11.1355 \ldots)(\mathrm{cm}) \tag{A1}
\end{equation*}
\] \\
(b) Area \(=\frac{1}{2} \times 30 \times 11.1355 \ldots\) \\
(M1) \\
Award (M1) for correct substitution in area of triangle formula
\[
=167(167.032 \ldots)\left(\mathrm{cm}^{2}\right) \quad \text { (A1)(ft) }
\]
\end{tabular} \& (a)

(b) \& \[
$$
\begin{aligned}
& \mathrm{BC}=\sqrt{32^{2}-30^{2}} \\
& 11(\mathrm{~cm}) \\
& \text { case (i) } \\
& \text { Area }=\frac{1}{2} \times 30 \times 11 \\
& =165\left(\mathrm{~cm}^{2}\right) \\
& \text { case (ii) } \\
& =165\left(\mathrm{~cm}^{2}\right) \\
& \quad \text { treated as a }
\end{aligned}
$$

\] \& | (M1) |
| :--- |
| (A1) |
| seen, but correct) |
| (M1) |
| (working shown) |
| (A1)(ft) |
| (MO)(A0)(ft) |
| the answer 11 is ks awarded here) | <br>

\hline
\end{tabular}

Rounding of an exact answer to 3 significant figures should be accepted if performed correctly. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is not essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of $\pi$ and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.
eg, Chi-squared, correlation coefficient, mean

| Markscheme | Candidates' Scripts |  | Marking |
| :--- | :--- | :--- | :--- |
| Chi-squared | (a) 7.7 | (A2) |  |
| $7.68(7.67543 \ldots)$ | (A2) | (b) 7.67 |  |
|  | (c) 7.6 | (A1) |  |
|  | (d) 8 | (A1) |  |
|  | (e) 7 |  |  |
|  | (e) 7.66 | (A0) |  |
|  |  | (A0) |  |

Regression line


Maximum/minimum/points of intersection


## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final $\boldsymbol{A}$ mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp .


## Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final $\boldsymbol{A}$ mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

| Markscheme |  | Candidates' Scripts |  | Marking |
| :--- | :--- | :--- | :--- | :--- |
| (a) $37000 \mathrm{~m}^{2}$ | (A1) | (a) $36000 \mathrm{~m}^{2}$ | (A0) |  |
| (b) | $3200 \mathrm{~m}^{3}$ | (A1) | (b) $3200 \mathrm{~m}^{2}$ | (Incorrect answer so units not considered) |

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

## 8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1. (a) $\frac{3474000 \times \pi}{1000}$ (M1)(M1)

Note: Award (M1) for correct numerator and (M1) for dividing by 1000
OR equivalent, such as $\frac{3474000 \times 2 \times \pi}{2000}$ ie diameter.
Do not accept use of area formula ie $\pi r^{2}$.
$10913.89287 \ldots(\mathrm{~km})$
(A1)
(C3)
[3 marks]
(b) $10900(\mathrm{~km})$
(A1)(ft)
Note: Follow through from part (a).
(c) $1.09 \times 10^{4}$
(A1)(ft)(A1)(ft)
Note: Follow through from part (b) only. Award (A1)(ft) for 1.09, and (A1)(ft) $\times 10^{4}$. Award ( $\mathbf{A O} \mathbf{O}(\mathbf{A O})$ for answers of the type: $10.9 \times 10^{3}$.
2. (a) $y=15.5 x-80$
(A1)(A1)
(C2)
Note: Award (A1) for $15.5 x$; (A1) for -80 . Award at most (A1)(A0) if answer is not an equation. Award (A0)(A1)(ft) for $y=-80 x+15.5$.
(b)


Temperature, ${ }^{\circ} \mathrm{C}$
(A1)(A1)
(C2)

Note: Award (A1) for a straight line using a ruler passing through (20, 230); (A1) for correct $y$-intercept. If a ruler has not been used, award at most (AO)(A1).
(c) $a=10$ AND $b=30$
(A1)(A1)

Note: Accept [10,30] or $10 \leq x \leq 30$.
3. (a) $1200 \times 7.0208$

Note: Award (M1) for multiplying by 7.0208 .
8424.96 (DKK)
(A1)
(C2)
[2 marks]
(b) (i) $0.95 \times 3450$
(M1)
Note: Award (M1) for multiplying 3450 by 0.95 (or equivalent).

$$
3277.50(\mathrm{DKK}) \quad \text { (A1) }
$$

(C2)
Note: The answer must be given to two decimal places unless already penalized in part (a).
(ii) $\frac{3277.50}{7.0208}$
(M1)

Note: Follow through from part (b)(i). Award (M1) for dividing their part (b)(i) by 7.0208.
466.83 (USD)
(A1)(ft)
(C2)
Note: The answer must be given to two decimal places unless already penalized in parts (a) or (b)(i).
4. (a) The product of $a$ and $b$ is divisible by 9 if and only if (is equivalent to) $a$ is divisible by 9 and $b$ is divisible by 9 .
(A1)(A1)(A1)

## OR

The product of $a$ and $b$ is divisible by 9 if and only if $a$ and $b$ are both divisible by 9 .

$$
\begin{equation*}
(A 1)(A 1)(A 1) \tag{C3}
\end{equation*}
$$

## OR

If the product of $a$ and $b$ is divisible by 9 then $a$ is divisible by 9 and $b$ is divisible by 9 and if $a$ is divisible by 9 and $b$ is divisible by 9 then the product of $a$ and $b$ is divisible by 9 .
(A1)(A1)(A1)
Note: Award (A1) for "is equivalent to," or "if and only if" or "iff" between their simple and compound propositions, (A1) for "and" in a compound proposition, (A1) for positioning $s$ on one side and $p \wedge q$ on the other side of their "is equivalent to". For example, award (A1)(A1)(A0) for expressing $p \Leftrightarrow(s \wedge q)$ in words.
(b) Award (A1) for each correct column.

| $p$ | $q$ | $s$ | $p \wedge q$ | $s \Leftrightarrow(p \wedge q)$ |
| :---: | :--- | :--- | :---: | :---: |
| $T$ | $T$ | $T$ | T | T |
| $T$ | $T$ | F | T | F |
| $T$ | F | T | F | F |
| $T$ | F | F | F | T |
| F | T | T | F | F |
| F | T | F | F | T |
| F | F | T | F | F |
| F | F | F | F | T |

Note: Follow through from their $p \wedge q$ column to their $s \Leftrightarrow(p \wedge q)$ column.
(c) The last column contains both true and false values.
(R1)

## OR

The entries in the " $s$ " column are different from the entries in the " $p \wedge q$ " column.
5. (a)

U

(A1)(A1)
Note: Award (A1) for 18, 12 and 1 in correct place on Venn diagram, (A1) for 3,16 and 11 in correct place on Venn diagram.
(b) $85-(3+16+11+18+12+1+2)$

Note: Award (M1) for subtracting the sum of their values from 85 .

22
(A1)(ft)
(C2)
Note: Follow through from their Venn diagram in part (a).
If any numbers that are being subtracted are negative award (M1)(A0).
(c) $\frac{14}{35}$
$\left(\frac{2}{5}, 0.4,40 \%\right)$
(A1)(ft)(A1)(ft)

Note: Award (A1) for correct numerator; (A1) for correct denominator. Follow through from their Venn diagram.
6. (a) median
(b) (i) $18-12$
(A1)
Note: Award (M1) for correct quartiles seen.
6 (g) ..... (A1)
(ii) 125
(A1)
(C1) [3 marks]
(c) Cafeteria 2
(A1)
$75 \%>50 \%$ (do not meet the requirement)
(R1)

## OR

$25 \%<50 \%$ (meet the requirement)
(R1)

[^0]7. (a) $x-4.5$

Note: Accept $w=x-4.5$ OR width $=x-4.5$.
(b) $2 x+2(x-4.5)=111$ (or equivalent)
(A1)(ft)
Note: Follow through from their expression for the width from part (a).
(c) $4 x-9=111$ (or equivalent)
(M1)
Note: Award (M1) for correctly removing the parentheses and collecting $x$ terms. This may be seen in part (b).

$$
\begin{equation*}
(x=) 30 \quad \text { (A1)(ft) } \tag{C2}
\end{equation*}
$$

Note: Follow through from their equation from part (b) provided $x>4.5$.
[2 marks]
(d) $\left|\frac{25-30}{30}\right| \times 100 \%$
(M1)

Note: Award (M1) for correct substitution into the percentage error formula. Accept a method in two steps where " $\times 100$ " is implicit from their answer.
16.7 (\%) (16.6666...)
(A1)(ft)
(C2)
Note: Follow through from their answer in (c). Award at most (M1)(A0) for a final answer that is negative.
8. (a) (i) $-\frac{3}{4}$
(A1)(A1)
(C2)

Note: Award (A1) for $\frac{3}{4}(0.75)$; (A1) for correct sign.
Award (A1)(A0) for $\frac{-3}{4} x$.
(ii) $y=-\frac{3}{4} x+3$
(A1)(ft)
(C1)

Note: Follow through from their gradient in part (a)(i). The equation must be in the form $y=m x+c$.
(b) (i) $\frac{4}{3}$
(A1)(ft)
(C1)

Note: Follow through from their gradient in part (a)(i).
(ii) $1=\frac{4}{3} \times 2+c$
(M1)

OR
$y-1=\frac{4}{3}(x-2)$
(M1)

Note: Award (M1) for correct substitution of their gradient and coordinates of point $P$ into equation of a line.

$$
\begin{equation*}
y=\frac{4}{3} x-\frac{5}{3} \tag{C2}
\end{equation*}
$$

(A1)(ft)
Note: Follow through from their (b)(i). The equation must be in the form $y=m x+c$ unless already penalized in part (a)(ii).
9. (a) $\tan B=\frac{12}{9}$
(A1)(M1)
Note: Award (A1) for 12 seen, (M1) for correct substitution into $\tan$ (or equivalent). Accept equivalent methods, such as Pythagoras, to find BC and correct substitution into other trig ratios. If $\tan ^{-1}\left(\frac{16}{9}\right)$ seen award (AO)(M1)(AO).
$53.1^{\circ}\left(53.1301 \ldots{ }^{\circ}\right)$
(A1)
(C3)
Note: If radians are used the answer is $0.927295 \ldots$; award at most (A1)(M1)(AO).
[3 marks]
(b) $30 \sin 70^{\circ}+4$
(M1)(M1)
Note: Award (M1) for $\sin 70^{\circ}=\frac{x}{30}$ (or equivalent) and (M1) for adding 4.
$32.2(32.1907 \ldots)(\mathrm{m}) \quad$ (A1)
(C3)
Note: If radians are used the answer is 27.2167...; award at most (M1)(M1)(A0).
[3 marks]
Total [6 marks]
10. (a) $x=3.5$

Note: Award (A1) for $x=$ a constant, (A1) for 3.5 seen.
(A1)(A1)
(C2)
(b) - 1 AND 8

Note: Award (AO)(A1)(ft) for " -8 and 1 ".
[2 marks]
(c) $\quad-4=a(0+1)(0-8)$
(M1)
Note: Award (M1) for correct substitution of their values from part (b) and substitution of $(0,-4)$ in $f(x)$.

$$
\begin{equation*}
(a=) \frac{1}{2} \tag{C2}
\end{equation*}
$$

(A1)(ft)

Note: Follow through from part (b).
11. (a) $A=\{3,6,9,12\}$ AND $B=\{2,4,6,8,10,12,14\}$

Note: Award (M1) for listing all elements of sets $A$ and $B$. May be seen in part (b). Condone the inclusion of 15 in set $A$ when awarding the (M1).

6, 12
(A1)(A1)
(C3)
Note: Award (A1) for each correct element. Award (A1)(A0) if one additional value seen. Award (AO)(A0) if two or more additional values are seen.
(b) (i) 3,9
(A1)(ft)(A1)(ft)
Note: Follow through from part (a) but only if their $A$ and $B$ are explicitly listed. Award (A1)(ft) for each correct element. Award (A1)(A0) if one additional value seen. Award (A0)(A0) if two or more additional values are seen.
(ii) 2
(A1)(ft)
Note: Follow through from part (b)(i).
12. (a) (i) $p=10$
(A1)
(C1)
(ii) $\quad q=56$
(A1)
(C1)
Note: Award (A1) for each correct value.
[2 marks]
(b)

(A1)(A1)
(C2)
Note: Award (A1)(ft) for their 3 correctly plotted points; award (A1)(ft) for completing diagram with a smooth curve through their points. The second (A1)(ft) can follow through from incorrect points, provided the gradient of the curve is never negative. Award (C2) for a completely correct smooth curve that goes through the correct points.
(c) a straight vertical line drawn at 35 (accept $35 \pm 1$ )

26 (students)
(A1)
(C2)
Note: Accept values between 25 and 27 inclusive.
13. (a) $55=a \times 2^{0}+10$

Note: Award (M1) for correct substitution of zero and 55 into the function.

45
(C2)
[2 marks]
(b) $45 \times 2^{-t}+10 \leq 20$

Note: Award (M1) for comparing correct expression involving 20 and their 45. Accept an equation.

$$
t=2.17(2.16992 \ldots) \quad \text { (A1)(ft) }
$$

(C2)
Note: Follow through from their part (a), but only if positive.
Answer must be in years; do not accept months for the final (A1).
(c) $m=) 10$
because as the number of years increases the number of turtles
approaches 10 $\quad$ (A1)
[2 marks]

Note: Award (R1) for a sketch with an asymptote at approximately $y=10$,
OR for table with values such as 10.003 and 10.001 for $t=14$ and $t=15$, for example,
OR when $t$ approaches large numbers $y$ approaches 10 .
Do not award (A1)(R0).

Total [6 marks]
14. (a) (i)

(A1)
(C1)
Note: Award (A1) for vertical line drawn at the mean (3.22 does not have to be seen) and correct region shaded.
(ii) 4.90
(A1)
(b) $0.323(0.323499 \ldots ; 32.3 \%)$
(A2)
Note: If final answer is incorrect, (M1)(A0) may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement " $\mathrm{P}(2 \leq X \leq 3)$ " (accept other variables for $X$ or "price" and strict inequalities).

(c) $2.51(2.51303 \ldots)$
(A2)
Note: If final answer is incorrect, (M1)(AO) may be awarded for correct shaded area shown on a sketch, below, or for a correct probability statement " $\mathrm{P}(X \leq a)=0.2$ " (accept other variables and strict inequality).

15. (a) -120 (AUD)
(A1)
(C1)
[1 mark]
(b) $-\frac{3}{5} x^{2}+14 x$
(A1)(A1)
(C2)

Note: Award (A1) for each correct term. Award at most (A1)(A0) for extra terms seen.
[2 marks]
(c) $-\frac{3}{5} x^{2}+14 x=0$
(M1)

Note: Award (M1) for equating their derivative to zero.
OR
sketch of their derivative (approximately correct shape) with $x$-intercept seen (M1)
$23 \frac{1}{3}\left(23.3,23.3333 \ldots, \frac{70}{3}\right)$
(A1)(ft)

Note: Award (C2) for $23 \frac{1}{3},\left(23.3,23.3333 \ldots, \frac{70}{3}\right)$ seen without working.

23
Note: Follow through from part (b).


[^0]:    Note: Do not award (A1)(R0). Award the (R1) for a correct comparison of percentages for both cafeterias, which may be in words. The percentage values or fractions must be seen. It is possible to award (A0)(R1).

