

# **Markscheme**

**May 2019**

**Mathematical studies**

**Standard level**

**Paper 2**

26 pages

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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- Working crossed out by the candidate should not be awarded any marks.
- Where candidates have written two solutions to a question, only the first solution should be marked.
- If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

**Example:**

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final <b>(A1)</b> (ignore the further working)
2.	$(x-6)(x+1)$	$x=6$ and $-1$	Do <b>not</b> award the final <b>(A1)</b>

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ <b>(M1)</b> Award <b>(M1)</b> for correct substitution in gradient formula $= -\frac{6}{5}$ <b>(A1)</b>	(i) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> Gradient is $= -\frac{6}{5}$ <b>(A1)</b> <i>(There is clear understanding of the gradient.)</i> $y = -\frac{6}{5}x + 9$ (ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> $y = -\frac{6}{5}x + 9$ <b>(A0)</b> <i>(There is confusion about what is required.)</i>	

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with **‘(ft)’**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate’s use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates’ Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b>  Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p><b>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the same question part.)</b></p>	<p><b>(M1)(A0)</b>  <i>(use of sine rule but with wrong values)</i></p> <p><b>(A0)</b></p>
<p>(b) <math>x = 7 \tan (22.0243\dots^\circ)</math> <b>(M1)</b>  <math>= 2.83</math> (2.83163...) <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math>  <math>= 6.26</math>  <b>but</b> case (ii) <math>6.26</math></p>	<p><b>(M1)</b>  <b>(A1)(ft)</b>  <b>(G0)</b>  <i>since no working shown</i></p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.  
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b> Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math></p> <p><b>Note: G marks are used only if no working has been shown and the answer is correct.</b></p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p> <p><b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.  
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
the 3 significant figure answer worked through from full calculator display;  
the exact value  $\left( \text{for example } \frac{2}{3} \text{ if applicable} \right)$ ;  
the full calculator display in the form 2.83163... as in the example above.  
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (e.g. complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ ;  $-p$ .  
 $p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

Significance level may be written as  $\alpha$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.
2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would <b>round to</b> the correct 3 sf)	<b>Correct</b> to 3 sf	<b>Incorrect</b> to 3 sf	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(A1)</b> irrespective of correct or incorrect rounding					
Unrounded answer not seen <sup>2</sup>	<b>(A1)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. <sup>3</sup>			

**Examples:**

<b>Markscheme</b>	<b>Candidates' Scripts</b>	<b>Marking</b>
9.43 (9.43398...) <b>(A1)</b>	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded)	<b>(A1)</b>
	(ii) 9.43398... is seen followed by 9.433; 9.44 <i>etc.</i> (incorrectly rounded)	<b>(A1)</b>
	(iii) 9.4	<b>(A1)</b>
	(iv) 9	<b>(A0)</b> (correct to 1 sf)
	(v) 9.3	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 9.44	<b>(A0)</b> (incorrectly rounded to 3 sf)

<b>Markscheme</b>	<b>Candidates' Scripts</b>	<b>Marking</b>
7.44 (7.43798...) <b>(A1)</b>	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded)	<b>(A1)</b>
	(ii) 7.43798... is seen followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	<b>(A1)</b>
	(iii) 7.4	<b>(A1)</b>
	(iv) 7	<b>(A0)</b> (correct to 1 sf)
	(v) 7.5	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 7.43	<b>(A0)</b> (incorrectly rounded to 3 sf)



**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in Pythagoras' formula</p> <p><math>= 11.1 (\sqrt{124}, 11.1355...)(\text{cm})</math> <b>(A1)</b></p> <p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in area of triangle formula</p> <p><math>= 167(167.032...)(\text{cm}^2)</math> <b>(A1)(ft)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  11 (cm) <b>(A1)</b> <i>(2 sf answer only seen, but correct)</i></p> <p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b> <i>(working shown)</i>  <math>= 165 (\text{cm}^2)</math> <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 (\text{cm}^2)</math> <b>(M0)(A0)(ft)</b> <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i></p>	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	<b>(G2)</b>
7.68 (7.67543...) <b>(A2)</b>	(b) 7.67	<b>(G1)</b>
	(c) 7.6	<b>(G1)</b>
	(d) 8	<b>(G0)</b>
	(e) 7	<b>(G0)</b>
	(e) 7.66	<b>(G0)</b>

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ <b>(A2)</b> $(y = 0.887686...x + 13.4895...)$ If an answer is not in the form of an equation award at most <b>(A1)(A0)</b> .	(a) $y = 0.89x + 13$ (b) $y = 0.88x + 13$ (c) $y = 0.88x + 14$ (d) (i) $y = 0.9x + 13$ (ii) $y = 0.8x + 13$ (e) $0.88x + 13$	<b>(G2)</b> <i>(both accepted)</i> <b>(G1)</b> <i>(one rounding error)</i> <b>(G1)</b> <i>(rounding error repeated)</i> <b>(G1)</b> <i>(1 sf not accepted)</i> <b>(G0)</b> <i>(one rounding error and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ <b>(A1)(A1)</b> $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$ (b) $(2.0, 4.4)$ (c) $(2.06, 4.4)$ (d) $(2, 4.4)$	<b>(A1)(A1)</b> <i>(both accepted)</i> <b>(A1)</b> <i>(same rounding error twice)</i> <b>(A1)</b> <i>(one rounding error)</i> <b>(A0)</b> <i>(1sf not accepted, one rounding error)</i>

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (eg,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ , ) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect answer so units not considered)</i>
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect units)</i>

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) ( $H_0$ :) (choice of) language is independent of gender (A1)

**Note:** Accept “there is no association between language (choice) and gender”.  
Accept “language (choice) is not dependent on gender”. Do not accept  
“not related” or “not correlated” or “not influenced”.

[1 mark]

- (b) 2 (A1)

[1 mark]

- (c) (i) 16.4 (16.4181...) (G1)

- (ii)  $\chi^2_{\text{calc}} = 8.69$  (8.68507...) (G2)

[3 marks]

- (d) (we) reject the null hypothesis (A1)(ft)  
8.68507... > 5.99 (R1)(ft)

**Note:** Follow through from part (c)(ii). Accept “do not accept” in place of “reject.” Do not  
award (A1)(ft)(R0).

**OR**

- (we) reject the null hypothesis (A1)  
0.0130034 < 0.05 (R1)

**Note:** Accept “do not accept” in place of “reject.” Do not award (A1)(ft)(R0).

[2 marks]

continued...

Question 1 continued

(e) (i)  $\frac{88}{110} \left( \frac{4}{5}, 0.8, 80\% \right)$  (A1)(A1)(G2)

**Note:** Award (A1) for correct numerator, (A1) for correct denominator.

(ii)  $\frac{88}{110} \times \frac{87}{109}$  (M1)(M1)

**Note:** Award (M1) for multiplying two fractions. Award (M1) for multiplying their correct fractions.

OR

$\left( \frac{46}{110} \right) \left( \frac{45}{109} \right) + 2 \left( \frac{46}{110} \right) \left( \frac{42}{109} \right) + \left( \frac{42}{110} \right) \left( \frac{41}{109} \right)$  (M1)(M1)

**Note:** Award (M1) for correct products; (M1) for adding 4 products.

$0.639 \left( 0.638532..., \frac{348}{545}, 63.9\% \right)$  (A1)(ft)(G2)

**Note:** Follow through from their answer to part (e)(i).

(iii)  $1 - \frac{67}{110} \times \frac{66}{109}$  (M1)(M1)

**Note:** Award (M1) for multiplying two correct fractions. Award (M1) for subtracting their product of two fractions from 1.

OR

$\frac{43}{110} \times \frac{42}{109} + \frac{43}{110} \times \frac{67}{109} + \frac{67}{110} \times \frac{43}{109}$  (M1)(M1)

**Note:** Award (M1) for all correct products, (M1) for adding three products.

$0.631 \left( 0.631192..., 63.1\%, \frac{344}{545} \right)$  (A1)(G2)

[8 marks]

Total [15 marks]

2. (a)  $AC^2 = 185^2 + 250^2 - 2 \times 185 \times 250 \times \cos(125^\circ)$  (M1)(A1)

**Note:** Award (M1) for substitution in the cosine formula; (A1) for correct substitution.

387 (387.015...) (m) (A1)(G2)

**Note:** If radians are used the answer is 154 (154.471...), award at most (M1)(A1)(A0).

[3 marks]

(b) (i)  $\frac{250}{\sin \hat{BAC}} = \frac{387.015...}{\sin(125^\circ)}$  (M1)(A1)(ft)

OR

$\cos^{-1}\left(\frac{185^2 + 387.015...^2 - 250^2}{2 \times 185 \times 387.015...}\right)$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution in the sine or cosine formulas; (A1)(ft) for correct substitution.

$\hat{BAC} = 31.9^\circ$  (31.9478...) (A1)(ft)(G2)

**Note:** Follow through from part (a).

(ii) (CAD =)  $53.1^\circ$  (53.0521...) (A1)(ft)

**Note:** Follow through from their part (b)(i) only if working seen.

[4 marks]

(c) (ACD =)  $70^\circ - (180^\circ - 125^\circ - 31.9478^\circ \dots)$  (M1)

**Note:** Award (M1) for subtracting their angle  $\hat{ACB}$  from  $70^\circ$ .

OR

(ADC =)  $360 - (85 + 70 + 125) = 80$

(ACD =)  $180 - 80 - 53.0521\dots$  (M1)

$46.9^\circ$  (46.9478...) (A1)(ft)(G2)

**Note:** Follow through from part (b)(i).

[2 marks]

continued...

Question 2 continued

$$(d) \quad \frac{185 \times 250 \times \sin(125^\circ)}{2} + \frac{287 \times 387.015 \dots \times \sin(53.0521 \dots^\circ)}{2} \quad (M1)(M1)(M1)$$

**Note:** Award **(M1)** for substitution in the area formula for either triangle; **(M1)** for correct substitution for both areas; **(M1)** for adding their two areas;

$$18942.8 \dots + 44383.9 \dots$$

$$63300 \text{ (m}^2\text{)} \text{ (63326.8} \dots \text{ (m}^2\text{))}$$

**(A1)(ft)(G3)**

**Note:** Follow through from parts (a) and (b)(ii).

**OR**

$$DC = \frac{287 \times \sin(53.0521 \dots)}{\sin(46.9478 \dots)} = 313.884 \dots$$

$$0.5 \times 287 \times 185 \times \sin 85^\circ + 0.5 \times 250 \times 313.884 \dots \times \sin 70^\circ$$

**M1M1M1**

**Note:** Award **(M1)** for substitution in the area formula for either triangle; **(M1)** for correct substitution for both areas; **(M1)** for adding their two areas.

$$26446.4 \dots + 36869.3 \dots$$

$$63300 \text{ (63315.8} \dots \text{) (m}^2\text{)}$$

**(A1)(ft)(G3)**

**[4 marks]**

**Total [13 marks]**

3. (a)  $\frac{\pi(5.2)^2 \times 13}{3}$  (M1)

**Note:** Award (M1) for correct substitution in the volume formula for cone.

368 (368.110...) cm<sup>3</sup> (A1)(G2)

**Note:** Accept 117.173...π cm<sup>3</sup> or  $\frac{8788}{75}\pi$  cm<sup>3</sup>.

[2 marks]

(b) (slant height)<sup>2</sup> = (5.2)<sup>2</sup> + 13<sup>2</sup> (M1)

**Note:** Award (M1) for correct substitution into the formula.

14.0 (14.0014...) (cm) (A1)(G2)

[2 marks]

(c) 14.0014... × (5.2) × π + (5.2)<sup>2</sup> × π (M1)(M1)

**Note:** Award (M1) for their correct substitution in the curved surface area formula for cone; (M1) for adding the correct area of the base. The addition must be explicitly seen for the second (M1) to be awarded. Do not accept rounded values here as may come from working backwards.

313.679... (cm<sup>2</sup>) (A1)

**Note:** Use of 3 sf value 14.0 gives an unrounded answer of 313.656...

314 (cm<sup>2</sup>) (AG)

**Note:** Both the unrounded and rounded answers must be seen for the final (A1) to be awarded.

[3 marks]

(d)  $2 \times \pi \times (5.2) \times h + 2 \times \pi \times (5.2)^2 = 314$  (M1)(M1)(M1)

**Note:** Award (M1) for correct substitution in the curved surface area formula for cylinder; (M1) for adding two correct base areas of the cylinder; (M1) for equating their total cylinder surface area to 314 (313.679...). For this mark to be awarded the areas of the two bases must be added to the cylinder curved surface area and equated to 314. Award at most (M1)(M0)(M0) for cylinder curved surface area equated to 314.

(h =) 4.41 (4.41051...) (cm) (A1)(G3)

[4 marks]

continued...



Question 3 continued

(e)  $\pi \times (5.2)^2 \times 4.41051\dots$  (M1)

**Note:** Award (M1) for correct substitution in the volume formula for cylinder.

375 (374.666...) (cm<sup>3</sup>) (A1)(ft)(G2)

**Note:** Follow through from part (d).

375 (cm<sup>3</sup>) > 368 (cm<sup>3</sup>) (R1)(ft)

OR

“volume of cylinder is larger than volume of cone” or similar (R1)(ft)

**Note:** Follow through from their answer to part (a). The verbal statement should be consistent with their answers from parts (e) and (a) for the (R1) to be awarded.

replace with the cylinder containers (A1)(ft)

**Note:** Do not award (A1)(ft)(R0). Follow through from their incorrect volume for the cylinder in this question part but only if substitution in the volume formula shown.

[4 marks]

Total [15 marks]

4. (a) (i) (amount taken in the 7<sup>th</sup> day):  $u_1 + 6d = 21$  (A1)

(ii) (amount taken in the 11<sup>th</sup> day):  $u_1 + 10d = 29$  (A1)

**Note:** Accept  $u_1 + (7-1)d = 21$  and  $u_1 + (11-1)d = 29$ . The equations do not need to be simplified. They should be given in terms of  $u_1$  and  $d$  for the marks to be awarded.

[2 marks]

(b) ( $u_1 =$ ) 9 (A1)(ft)

( $d =$ ) 2 (A1)(ft)

**Note:** Follow through from part (a), but only if values are positive and  $u_1 < 21$ .

[2 marks]

(c) ( $S_{30} =$ )  $\frac{30}{2}(2 \times 9 + (30-1) \times 2)$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution in the sum of an arithmetic sequence formula; (A1)(ft) for their correct substitution.

1140 (mg) (A1)(ft)(G3)

**Note:** Follow through from their  $u_1$  and  $d$  from part (b).

[3 marks]

(d) (i)  $20 \times (0.5)^4$  (M1)(A1)

**Note:** Award (M1) for substitution into the geometric sequence formula, (A1) for correct substitution.

1.25 (mg) (A1)(G3)

continued...

Question 4 continued

(ii)  $20 \times (0.5)^{k-1} < 0.06$  (M1)(M1)

**Note:** Award **(M1)** for correct substitution into the geometric sequence formula; **(M1)** for comparing their expression to 0.06. Accept an equation instead of inequality.

$(k =) 10$  (10 th day) (A1)(ft)(G3)

**Note:** Follow through from part (d)(i), if  $0 < r < 1$ . Follow through answers must be rounded up for final mark.

(iii)  $\frac{20(1 - 0.5^{10})}{1 - 0.5}$  (M1)(A1)(ft)

**Note:** Award **(M1)** for substitution into sum of a geometric sequence formula, **(A1)(ft)** for correct substitution.  
Follow through from their  $u_1$  and  $r$  in part (d)(i), if  $0 < r < 1$ . Follow through from their  $k$  in part (d)(ii) but only if  $k$  is a positive integer.

40.0 (39.9609...) (mg) (A1)(ft)(G2)

**[9 marks]**

**Total [16 marks]**

5. (a)  $\frac{1}{3} \times 2^3 + \frac{3}{4} \times 2^2 - 2 - 1$  (M1)

**Note:** Award (M1) for correct substitution into function.

$2.67 \left( \frac{8}{3}, 2.66666... \right)$  (A1)(G2)

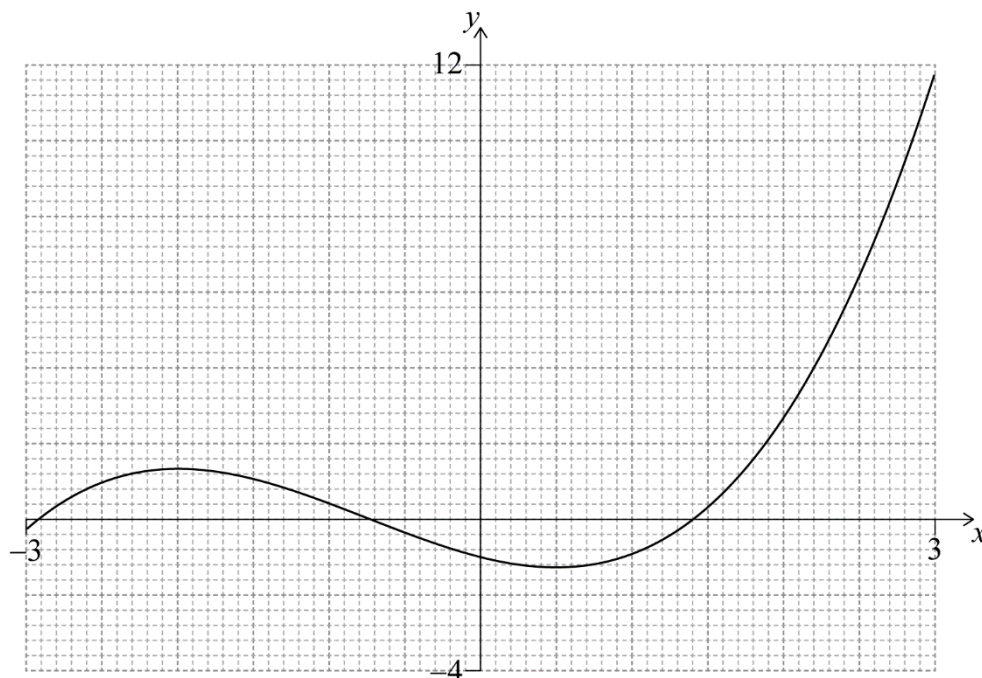
[2 marks]

(b)  $-1$  (A1)

**Note:** Accept  $(0, -1)$ .

[1 mark]

(c)



(A1)(A1)(A1)(A1)

**Note:** Award (A1) for correct window and axes labels,  $-3$  to  $3$  should be indicated on the  $x$ -axis and  $-4$  to  $12$  on the  $y$ -axis.

(A1) for smooth curve with correct cubic shape;

(A1) for  $x$ -intercepts: one close to  $-3$ , the second between  $-1$  and  $0$ , and third between  $1$  and  $2$ ; and  $y$ -intercept at approximately  $-1$ ;

(A1) for local minimum in the 4th quadrant and maximum in the 2nd quadrant, in approximately correct positions.

Graph paper does not need to be used. If window not given award at most (A0)(A1)(A0)(A1).

[4 marks]

continued...

Question 5 continued

(d)  $x^2 + \frac{3}{2}x - 1$  (A1)(A1)(A1)

**Note:** Award (A1) for each correct term. Award at most (A1)(A1)(A0) if there are extra terms.

[3 marks]

(e)  $2^2 + \frac{3}{2} \times 2 - 1$  (M1)

**Note:** Award (M1) for correct substitution of 2 in their derivative of the function.

6 (A1)(ft)(G2)

**Note:** Follow through from part (d).

[2 marks]

(f)  $\frac{8}{3} = 6(2) + c$  (M1)

**Note:** Award (M1) for 2, their part (a) and their part (e) substituted into equation of a straight line.

$$c = -\frac{28}{3}$$

OR

$\left(y - \frac{8}{3}\right) = 6(x - 2)$  (M1)

**Note:** Award (M1) for 2, their part (a) and their part (e) substituted into equation of a straight line.

OR

$y = 6x - \frac{28}{3}$  ( $y = 6x - 9.33333...$ ) (M1)

**Note:** Award (M1) for their answer to (e) and intercept  $-\frac{28}{3}$  substituted in the gradient-intercept line equation.

$-18x + 3y + 28 = 0$  (accept integer multiples) (A1)(ft)(G2)

**Note:** Follow through from parts (a) and (e).

[2 marks]

continued...

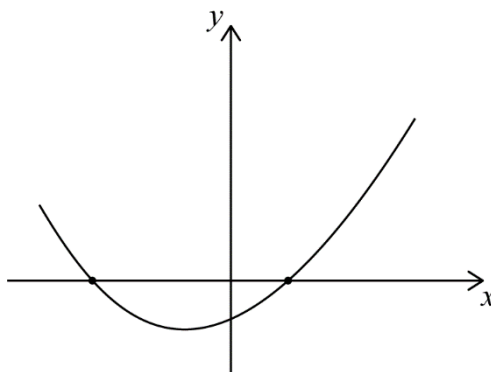
Question 5 continued

(g)  $x^2 + \frac{3}{2}x - 1 = 0$

(M1)

**Note:** Award (M1) for setting their derivative = 0.

OR



(M1)

**Note:** Award (M1) for a sketch of the derivative function with roots in approximately correct place.

$p = -2$

(A1)(ft)

$q = 0.5$

(A1)(ft)

**Note:** Accept “Function has a local maximum at  $x = -2$  and local minimum at  $x = 0.5$ ” instead of  $p = -2$  and  $q = 0.5$ . Follow through from their derivative in part (d), only if their  $p$  and  $q$  values are between  $-3$  and  $3$ . Award at most (M1)(A0)(A1)(ft) if working is shown leading to an answer of  $p = 0.5$  and  $q = -2$ .

The derivative must be used for (M1) to be awarded. Award at most (G1) for correct values of  $p$  and  $q$  without working OR a sketch of the function shown with max and min points identified, and correct values of  $p$  and  $q$ . Award (G0) for  $0.5$  and  $-2$  (given without specifying  $p$  and  $q$ ). Award (G0) if coordinates of max and min are given instead of  $p$  and  $q$  values.

[3 marks]

continued...

Question 5 continued

(h)  $-1.27 \leq f(x) \leq 1.33$

$$\left( -1.27083... \leq f(x) \leq 1.33333..., -\frac{61}{48} \leq f(x) \leq \frac{4}{3} \right) \quad (\mathbf{A1(ft)})(\mathbf{A1})(\mathbf{ft})(\mathbf{A1})$$

**Note:** Award **(A1)** for  $-1.27$  seen, **(A1)** for  $1.33$  seen, and **(A1)** for correct weak inequalities with **their** endpoints in the correct order. For example, award **(A0)(A0)(A0)** for answers like  $5 \leq f(x) \leq 2$ . Accept  $y$  in place of  $f(x)$ . Accept alternative correct notation such as  $[-1.27, 1.33]$ .

Follow through from their  $p$  and  $q$  values from part (g) only if their  $f(p)$  and  $f(q)$  values are between  $-4$  and  $12$ . Award **(A0)(A0)(A0)** if their values from (g) are given as the endpoints.

**[3 marks]**

**Total [20 marks]**

6. (a)  $950 \times \left(1 + \frac{5}{12 \times 100}\right)^{12 \times 3}$  (M1)(A1)

**Note:** Award (M1) for substitution in the compound interest formula: (A1) for correct substitution.

**OR**

$$N = 3$$

$$I\% = 5$$

$$PV = 950$$

$$P/Y = 1$$

$$C/Y = 12$$

(A1)(M1)

**Note:** Award (A1) for  $C/Y = 12$  seen, (M1) for other correct entries.

**OR**

$$N = 36$$

$$I\% = 5$$

$$PV = 950$$

$$P/Y = 12$$

$$C/Y = 12$$

(A1)(M1)

**Note:** Award (A1) for  $C/Y = 12$  seen, (M1) for other correct entries.

1103.40 (EUR)

(A1)(G3)

**Note:** Answer must be given to 2 decimal places.

[3 marks]

(b)  $(20 \times 3 + 1100) - 1103.40$  (M1)(M1)

**Note:** Award (M1) for correct substitution into cost of bike function, (M1) for subtracting their answer to part (a). This subtraction may be implied by their final answer (follow through from their part (a) for this implied subtraction).

56.60 (EUR)

(A1)(ft)(G3)

**Note:** Follow through from part (a). The answer must be two decimal places.

[3 marks]

continued...



Question 6 continued

(c) **METHOD 1**

$$950 \times \left(1 + \frac{5}{12 \times 100}\right)^{12x} = 20x + 1100 \quad (\text{M1})(\text{M1})$$

**Note:** Award **(M1)** for their correct substitution in the compound interest formula with a variable in the exponent; **(M1)** for comparing their expressions provided variables are the same (not an expression with  $x$  for years and another with  $x$  representing months). Award at most **(M0)(M1)(A0)(M1)(A0)** for substitution of an integer in both expressions and comparison of the results. Accept inequality.

$$\begin{aligned} (x =) 4.52157... \text{ (years)} & \quad (\text{A1})(\text{ft}) \\ 4.52157... \times 12 (= 54.2588...) & \quad (\text{M1}) \end{aligned}$$

**Note:** Award **(M1)** for multiplying **their** value for  $x$  by 12. This may be implied.

$$m = 55 \text{ (months)} \quad (\text{A1})(\text{ft})(\text{G4})$$

**METHOD 2**

$$950 \times \left(1 + \frac{5}{12 \times 100}\right)^m = 20 \times \frac{m}{12} + 1100 \quad (\text{M1})(\text{M1})(\text{M1})$$

**Note:** Award **(M1)** for their correct substitution in the compound interest formula with a variable in the exponent to solve; **(M1)** for comparing their expressions provided variables are the same; **(M1)** for converting years to months in these expressions. Award at most **(M0)(M1)(A0)(M1)(A0)** for substitution of an integer in both expressions and comparison of the results. Accept inequality.

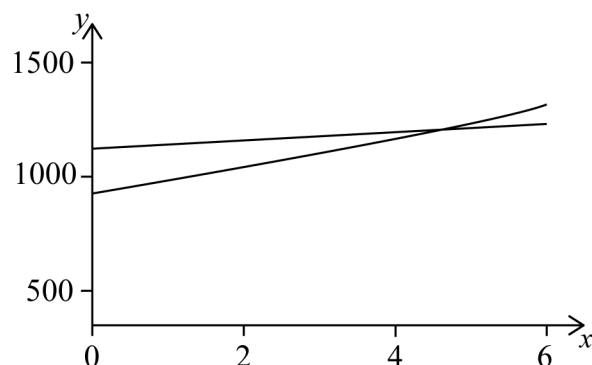
$$\begin{aligned} m = 54.2588... \text{ (months)} & \quad (\text{A1})(\text{ft}) \\ m = 55 \text{ (months)} & \quad (\text{A1})(\text{ft})(\text{G4}) \end{aligned}$$

**[5 marks]**

continued...

Question 6 continued

**METHOD 3**



(M1)(M1)

**Note:** Award (M1) for each graph drawn.

( $x =$ ) 4.52157... (years)

(A1)(ft)

4.52157...  $\times$  12 (= 54.2588...)

(M1)

**Note:** Award (M1) for multiplying **their** value for  $x$  by 12. This may be implied.

If the graphs drawn are in terms of months, leading to a value of 54.2588..., award (M1)(M1)(M1)(A1), consistent with METHOD 2.

$m = 55$  (months)

(A1)(ft)(G4)

**Note:** Follow through for a compound interest formula consistent with their part (a). The final (A1)(ft) can only be awarded for correct answer, or **their** correct answer following through from previous parts and only if value is rounded up. For example, do not award (M0)(M0)(A0)(M1)(A1)(ft) for an unsupported “5 years  $\times$  12 = 60” or similar.

**Total [11 marks]**